Die Funktion $\quad f(x) = -3x^3 + 12x^2 - 12x$

1. *Nullstellen:*
   Bed.: $f(x) = 0$

$$\begin{aligned} -3x^3 + 12x^2 - 12x &= 0 \qquad | : (-3) \\ x^3 - 4x^2 + 4x &= 0 \\ x\,(x^2 - 4x + 4) &= 0 \\ x_1 &= 0 \end{aligned}$$

$$\begin{aligned} x^2 - 4x + 4 &= 0 \\ x_{2/3} &= 2 \pm \sqrt{4-4} \\ x_{2/3} &= 2 \end{aligned}$$

2. *Steigungen in den Nullstellen:*

$$\begin{aligned} f'(x) &= -9x^2 + 24x - 12 \\ f'(0) &= -12 \\ f'(2) &= 0 \end{aligned}$$

3. *Extrema:*
   notwendige Bed.: $f'(x) = 0$

$$\begin{aligned} -9x^2 + 24x - 12 &= 0 \qquad | : (-9) \\ x^2 - \frac{8}{3}x + \frac{4}{3} &= 0 \\ x_{1/2} &= \frac{4}{3} \pm \sqrt{\frac{16}{9} - \frac{12}{9}} \\ x_{1/2} &= \frac{4}{3} \pm \frac{2}{3} \\ x_1 = 2 & \qquad x_2 = \frac{2}{3} \end{aligned}$$

$Max(2 \mid 0) \qquad Min(\frac{2}{3} \mid -\frac{32}{9})$

Minimum oder Maximum?

$$\begin{aligned} f''(x) &= -18x + 24 \\ f''(2) &= -36 + 24 < 0 \\ f''(\tfrac{2}{3}) &= -12 + 24 > 0 \end{aligned}$$

4. *Gleichung der Tangente* *im Punkt* $W(\frac{4}{3} \mid ?)$:

$$\begin{aligned} f(\tfrac{4}{3}) &= -\frac{16}{9} \\ m &= f'(\tfrac{4}{3}) = 4 \\ y &= 4x + b \\ -\frac{16}{9} &= 4 \cdot \frac{4}{3} + b \implies b = -7\frac{1}{9} \\ y &= 4x - 7\frac{1}{9} \end{aligned}$$

Graph: